

# Empirical Features of Congested Traffic States and Their Implications for Traffic Modeling

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We investigate characteristic properties of the congested traffic states on a 30 km long stretch of the German freeway A5 north of Frankfurt/Main. Among the approximately 245 breakdowns of traffic flow in 165 days, we have identified five different kinds of spatio-temporal congestion patterns and their combinations. Based on an “adaptive smoothing method” for the visualization of detector data, we also discuss particular features of breakdowns such as the “boomerang effect” which is a sign of linearly unstable traffic flow. Controversial issues such as “synchronized flow” or stop-and-go waves are addressed as well. Finally, our empirical results are compared with different theoretical concepts and interpretations of congestion patterns, in particular first- and second-order macroscopic traffic models.

## 1 Summary of Previous Models and Empirical Results

Understanding traffic dynamics can not only help to identify reasons for bottlenecks. It also contributes to the development of modern driver and traffic assistance systems aiming at the improvement of safety, comfort, and capacity. Progress has been made by empirical studies and theoretical modelling approaches. Apart from traffic scientists, mathematicians and physicists have also recently contributed to these fields. Because

of the numerous publications, our introductory overview can only be selective, so that we refer the reader to some comprehensive reviews (e.g., Gerlough and Huber, 1975; Vumbaco, 1981; Leutzbach, 1988; May, 1990; Brilon *et al.*, 1993; Transportation Research Board, 1996; Gartner *et al.*, 1997; Helbing, 1997a; Daganzo, 1997a; Bovy, 1998; Hall, 1999; Brilon *et al.*, 1999; Chowdhury *et al.*, 2000b, with a focus on cellular automata; Helbing, 2001a, containing 800 references; Nagatani, 2002).

## 1.1 Modeling Approaches

The main modeling approaches can be classified as follows: *Car-following models* focus on the non-linear interaction and dynamics of single vehicles. They specify their acceleration mostly as a function of the distance to the vehicle ahead, the own and relative velocity (e.g., Reuschel, 1950a, b; Gazis *et al.*, 1959, 1961; May and Keller, 1967; Gipps, 1981; Gibson, 1981; Bando *et al.*, 1994, 1995a; Krauß, 1998; Treiber *et al.*, 2000; Brackstone and McDonald, 2000). *Submicroscopic models* take into account even details such as perception thresholds, changing of gears, acceleration characteristics of specific car types, reactions to brake lights and wipers (Wiedemann, 1974; Fellendorf, 1996; Ludmann *et al.*, 1997). In favour of numerical efficiency, *cellular automata* describe the dynamics of vehicles in a coarse-grained way by discretizing space and time (Cremer and Ludwig, 1986; Biham *et al.*, 1992; Nagel and Schreckenberg, 1992; Chowdhury *et al.*, 2000b). *Gas-kinetic models* agglomerate over many vehicles and formulate a partial differential equation for the spatio-temporal evolution of the vehicle density and the velocity distribution. While Boltzmann-like approaches (Prigogine and Andrews, 1960; Prigogine and Herman, 1971; Paveri-Fontana, 1975; Phillips, 1977, 1979a, b; Nelson, 1995; Helbing, 1995c) are mainly suitable for small densities, Enskog-like approaches (Helbing, 1995d, 1996b, 2001a; Wagner, 1997a; Klar and Wegener, 1997, 1999a, b; Helbing and Treiber, 1998a; Shvetsov and Helbing, 1999) take into account corrections due to finite space requirements of vehicles. The main application of gas-kinetic models is the theoretical derivation of *macroscopic traffic equations* for the vehicle density and average velocity. It is common to distinguish two classes of macroscopic models: *First-order models* such as the Lighthill-Whitham-Richard model (Lighthill and Whitham, 1955; Richards, 1956) or the Burgers equation (Burgers, 1974) are based on a partial differential equation for the density and a velocity-density relation or a fundamental diagram (flow-density relation) only. *Second-order models* contain an additional partial differential equation for the average velocity and take into account the finite relaxation time to adapt the velocity to changing traffic conditions (Payne, 1971, 1979; Whitham, 1974; Cremer, 1979; Papageorgiou, 1983; Kühne, 1984a; Smulders, 1986; Kerner and Konhäuser, 1993; Helbing, 1995b; Helbing and Treiber, 1998a; Lee *et al.*, 1998). If identical driver-vehicle units are assumed, macroscopic traffic models can be also directly derived from microscopic car-following models (Payne, 1971, 1979a; Nelson, 2000; Helbing *et al.*, 2002), so that the approximations required in gas-kinetic derivations can be avoided. This is particularly interesting for a simultaneous micro-macro-simulation

(Helbing *et al.*, 2002), which can be performed on-line based on empirical boundary conditions. Finally, *mesoscopic* or *hybrid traffic models* describe the dynamics of single vehicles in response to aggregate quantities such as the density (Wiedemann and Schwerdtfeger, 1987; Schwerdtfeger, 1987; Kates *et al.*, 1998), while *queueing models* restrict to the temporal change of numbers of vehicles on larger street sections as a function of entering and leaving flows (Newell, 1982; Kerner, 2001), which is sufficient to determine the travel times (Helbing, 2003).

Each of the above mentioned modelling approaches has its own strengths and areas of applications. While cellular and macroscopic traffic models are presently in use for the identification and forecast of the traffic state between measurement cross sections (Nagel, 1996b, 1998; Esser and Schreckenberg, 1997; Esser *et al.*, 1999; Kaumann *et al.*, 2000), microscopic car-following and submicroscopic models are applied to the development of driver and traffic assistance systems (Treiber and Helbing, 2001). In any case, for most practical applications it is not sufficient to reproduce the relation between flow and density. It is equally important to reproduce the various different traffic states formed on freeways. This will be the main focus of this paper.

## 1.2 Flow-Density Relation and Wide Data Scattering

Here, we will shortly summarize important facts required for the discussion in this paper: The empirical flow-density relation  $Q(\rho)$  depends on the location of the measurement section. Upstream of a bottleneck, it is discontinuous and looks comparable to a mirror image of the Greek letter lambda. The two branches of this reverse lambda are used to define free low-density and congested high-density traffic (see, e.g., Koshi *et al.*, 1983; Hall *et al.*, 1986; Neubert *et al.*, 1999b; Kerner, 2000a; cf. also Edie and Foote, 1958). At low vehicle densities, the relationship between flow and density is more or less one-dimensional. The slope  $Q'(\rho)$  at vanishing density  $\rho = 0$  veh./km corresponds to the maximum average speed  $V_0$  on the respective road section, but it decreases with increasing density  $\rho$  (due to mutual obstructions during attempted overtaking maneuvers).

After the flow-density relation  $Q(\rho) = \rho V(\rho)$  reaches a maximum at some critical density  $\rho = \rho_{\text{cr}}$  due to a decrease in the average vehicle speeds  $V(\rho)$ , there is a capacity drop (Edie, 1961; Treiterer and Myers, 1974; Ceder and May, 1976; Payne, 1984), which is related to an increase of the average (brutto and netto) time gaps (Banks, 1999; Neubert *et al.*, 1999; Tilch and Helbing, 2000; Nishinari *et al.*, 2003). This has inspired Hall *et al.* to relate traffic dynamics with catastrophe theory (Hall, 1987; Dillon and Hall, 1987; Persaud and Hall, 1989; Gilchrist and Hall, 1989). The flow-density data at higher densities are widely scattered in a two-dimensional area (see, e.g., Koshi *et al.*, 1983; Hall *et al.*, 1986; Leutzbach, 1988; Kühne, 1991b) and erratically varying (Kerner and Rehborn, 1996b) around the so-called jam line (Kerner and Konhäuser, 1994). It can be represented in the form

$$J(\rho) = \frac{1}{T} \left( 1 - \frac{\rho}{\rho_{\text{jam}}} \right), \quad (1)$$

where the parameter  $\bar{T}$  can be interpreted as the average netto time gap of vehicles leaving congested regions and  $\rho_{\text{jam}}$  as the density inside of (standing) traffic jams. The slope  $c = -1/(\rho_{\text{jam}}\bar{T})$  determines the propagation velocity of perturbations in congested traffic (Kerner, 1998a). Moreover, recent empirical investigations of single-vehicle data (see Fig. (1)) have shown that changes of congested flow  $Q$  with time  $t$  are very well approximated by

$$Q(t) = \frac{1}{T(t)} \left( 1 - \frac{\rho(t)}{\rho_{\text{jam}}(t)} \right), \quad (2)$$

and the scattering of “synchronized” congested traffic can be interpreted mainly as an effect of the large variation in the netto time gaps  $T$  (Nishinari *et al.*, 2003), as suggested by Banks (1999).

Figure 1: The two-dimensional scattering of empirical flow-density data in synchronized traffic flow of high density  $\rho \geq 45$  veh/km/lane, see (a), is well reproduced by the jam relation (1), when not only the variation of the density  $\rho$ , but also the empirically measured variation of the average time gap  $T$  and the maximum density  $\rho_{\text{max}}$  is taken into account as in Eq. (2), see (b) (from Nishinari *et al.*, 2003). The pure density-dependence  $J(\rho)$  (thick black line) is linear and cannot explain a two-dimensional scattering. However, variations of the average time gap  $T$  change its slope  $-1/(\rho_{\text{max}}T)$  (see arrows), and about 95% of the data are located between the thin lines  $J(\rho, \bar{T} \pm 2\Delta T, 1/l) = (1 - \rho l)/(\bar{T} \pm 2\Delta T)$ , where  $l = 3.6$  m is the average vehicle length,  $\bar{T} = 2.25$  s the average time gap, and  $\Delta T = 0.29$  s the standard deviation of  $T$ . The predicted form of this area is club-shaped, as demanded by Kerner (1998b).

In an intermediate density range with  $\rho_{c1} < \rho < \rho_{c2}$ , both free and congested traffic may be found at different times, which points to hysteresis (Treiterer and Myers, 1974). The traffic states corresponding to the tip of the inverse lambda-shaped flow-density relation can last for many minutes (see, e.g., Cassidy and Bertini, 1999), but they are not stable, since it is only a matter of time until a transition to the lower, congested branch of the lambda takes place (Persaud *et al.*, 1998; see also Elefteriadou *et al.*, 1995). The probability  $P(\rho)$  of a transition from free to congested traffic during a given time period is 0 for  $\rho = \rho_{c1}$  and becomes 1 at  $\rho = \rho_{c2} = \rho_{\text{cr}}$ . This indicates a metastability of traffic flow in the intermediate density range between  $\rho_{c1}$  and  $\rho_{c2}$  (Kerner and Rehborn, 1996b, 1997; Kerner, 1998b, 1999a, b, c, 2000a, b). The transition probability increases with growing density (Persaud *et al.*, 1998).

### 1.3 Stop-and-Go Waves

It is possible to distinguish several forms of congested traffic: Stop-and-go waves (start-stop waves) have been empirically studied by a lot of authors, including Edie and Foote

(1958), Mika *et al.* (1969), and Koshi *et al.* (1983). The latter have found that the parts of the velocity-profile which belong to the fluent stages of stop-and-go waves do not significantly depend on the flow (regarding their height and length), while their oscillation frequency does. Correspondingly, there is no *characteristic* frequency of stop-and-go traffic. The average duration of one wave period is normally between 4 and 20 minutes for wide traffic jams (see, e.g., Mika *et al.*, 1969; Kühne, 1987; Helbing, 1997a, c, e), and the average wave length between 2.5 and 5 km (see, e.g., Kerner, 1998a). Stop-and-go waves propagate against the direction of the vehicle flow (Edie and Foote, 1958; Mika *et al.*, 1969) with a velocity  $c = -15 \pm 5$  km/h (see, e.g., Mika *et al.*, 1969; Kerner and Rehborn, 1996a; Kerner, 2000a, b; Cassidy and Mauch, 2001) and without spreading (Cassidy and Windover, 1995; Windover, 1998; Muñoz and Daganzo, 1999; see also Kerner and Rehborn, 1996a, and the flow and speed data reported by Foster, 1962; Cassidy and Bertini, 1999). Moreover, wide moving jams, i.e. moving jams whose width in longitudinal direction is considerably higher than the width of the jam fronts, are characterized by stable wave profiles and by kind of “universal” parameters (Kerner and Rehborn, 1996a). Apart from the propagation velocity  $c$ , these include the density  $\rho_{\text{jam}}$  inside of jams, the average velocity and flow inside of traffic jams, both of which are approximately zero, and the outflow  $Q_{\text{out}}$  from jams. Therefore, fully developed traffic jams can move in parallel over long time periods and road sections through free traffic or “synchronized” congested flow (Kerner, 1998b). The propagation speed of wide jams is not even influenced by ramps or intersections (Kerner and Rehborn, 1996a; Kerner, 2000a, b). However, the concrete values of the characteristic parameters slightly depend on the accepted safe time clearances, the average vehicle length, truck fraction, and weather conditions (Kerner and Rehborn, 1998a).

## 1.4 Queued or “Synchronized” Traffic and “Pinch Effect”

The most common form of congestion is not localized like a wide moving jam, but spatially extended, and it often persists over several hours. In contrast to stop-and-go waves, the flow and velocity stay finite. Nevertheless, the speed is significantly reduced, and there is still some capacity drop (Banks, 1991; Kerner and Rehborn, 1996b, 1998b; Persaud *et al.*, 1998; Westland, 1998; Cassidy and Bertini, 1999; see also May, 1964; Persaud, 1986; Banks, 1989, 1990; Agyemang-Duah and Hall, 1991; Daganzo, 1996). As the downstream front of this form of congestion is fixed (e.g. at the location of an on-ramp), it is natural to interpret it as a queuing effect at bottlenecks, when the outflow from a road section is exceeded by the inflow, e.g. during rush hours or due to construction sites or accidents. While Kerner and Rehborn (1996b) call these extended forms of congested traffic “*synchronized flow*” (because of the synchronization of the velocities among lanes), Daganzo (2002b) speaks of 1-pipe flow. Note, however, that a synchronization of the velocity-profiles in neighboring lanes is found for *all* congested traffic states including stop-and-go waves or wide moving jams (Helbing, 1997a). This synchronization is caused by lane changing maneuvers which equilibrate speed differences

among neighboring lanes, when drivers feel obstructed by congested traffic (Shvetsov and Helbing, 1999; see also the Java simulation applet of two-lane traffic on a ring road at <http://www.mtreiber.de>).

Kerner and Rehborn (1996b) distinguish three different kinds of “synchronized” congested traffic:

- (i) *stationary and homogeneous states* where both the average speed and the flow rate are stationary during a relatively long time interval (see, e.g., also Hall and Agyemang-Duah, 1991; Persaud *et al.*, 1998; Westland, 1998),
- (ii) *“homogeneous-in-speed states”* (see also Kerner, 1998b; Lee *et al.*, 2000) reminding of recovering traffic (Treiber and Helbing, 1999; Helbing, 2001a), which are mainly found downstream of bottlenecks and are characterized by the fact that only the average vehicle speed is stationary, and
- (iii) *non-stationary and non-homogeneous states* (see also Kerner, 1998b; Cassidy and Bertini, 1999; Treiber *et al.*, 2000).

The mechanism for the formation of wide moving jams or stop-and-go waves is still controversial. Treiterer’s (1966, 1974) evaluations of vehicle trajectories based on aerial photographs suggested the existence of *“phantom traffic jams”*, i.e. the spontaneous formation of traffic jams with no obvious reason such as an accident or a bottleneck. However, the spontaneous appearance of stop-and-go traffic has recently been questioned. According to Daganzo (2002a), the breakdown of free traffic “can be traced back to a lane change in front of a highly compressed set of cars”, which shows that there is actually a reason for jam formation, although its origin can be a rather small disturbance. Daganzo (2002b) suggests in accordance with Cassidy and Mauch (2001) that small oscillations may grow in amplitude due to a pumping effect at ramps. In his empirical investigations, Kerner (1998a) finds that jams can be born from extended congested traffic, which is based on the “pinch effect”: Upstream of a section with homogeneous congested traffic close to a bottleneck, there is a so-called *“pinch region”* characterized by the spontaneous birth of small narrow density clusters, which are growing while they travel further upstream. Wide moving jams are eventually formed by the merging or disappearance of narrow jams. Once formed, wide jams seem to suppress the occurrence of new narrow jams in between. Similar findings were reported by Koshi *et al.* (1983), who observed that “ripples of speed grow larger in terms of both height and length of the waves as they propagate upstream”. Note that, instead of forming wide jams, narrow jams may coexist when their distance is larger than about 2.5 km (Kerner, 1998a; Treiber *et al.*, 2000).

## 1.5 Organisation of this Paper

The mechanism of jam formation will be one of the focusses of this paper. Section 2 will describe our measurement section and its topology, the traffic data, and how we evaluate them. In particular, we will propose and use an adaptive smoothing algorithm as an alternative to the use of cumulative plots (Newell, 1982; Cassidy and Windover, 1995; Coifman, 2002; Muñoz and Daganzo, 2002a; Bertini *et al.*, 2003). It allows us to obtain an intuitive picture of the traffic state on the entire freeway section. In Section 3, we will discuss representative examples of the traffic states and phenomena that we have identified in our data. Altogether, we will classify free traffic, five different congested traffic states (including the above mentioned ones), and combinations of them. In particular, we will present empirical support for the existence of growing perturbations, which trigger the breakdown of traffic flow. According to Sec. 4, this feature is not compatible with first-order macroscopic traffic models such as the Lighthill-Whitham-Richards model (Lighthill and Whitham, 1995; Richards, 1956), while it is reproduced by second-order models. We will also present a theory (and a phase diagram) of congested traffic states, which is compatible with our empirical findings and allows one to classify the rich variety of existing traffic models. Finally, Sec. 5 summarizes our results and discusses further research directions, e.g. whether it is possible to reach a synthesis of first- and second-order models.

## 2 Investigation Site and Data Evaluation

During the last year, we have collected data from various freeways around the world in order to study spatio-temporal phenomena in traffic, to check predictions of traffic models, and to calibrate their parameters. Most of the data from Germany, The Netherlands, Japan, and the US are aggregated data from measurement cross sections such as one-minute data for the average velocity and the vehicle flow, but we have also investigated single vehicle data, data from car-following experiments, and floating-car data. Here, we will focus on one-minute velocity data from a freeway section which frequently suffers from serious congestion and has also been investigated by Kerner (Kerner and Rehborn, 1996a, b, 1997; Kerner 1998a, b, 1999a, c, 2000a, b, c, 2002). Measurements were available for all freeway lanes and most ramps, but apart from the investigation of exceptional situations such as accidents, we have arithmetically averaged the speeds over all freeway lanes. A sketch of the measurement site is shown in Fig. 2. It shows an approximately 30 kilometer long freeway section of the German autobahn A5 near Frankfurt/Main, which has mostly 3 lanes into each direction, three intersections with other freeways, and one junction. At the intersections, there are additional merging and diverging lanes, some of which are more than 1 kilometer long. Between the intersections “Frankfurt North-West” and “Bad Homburg”, but also between the latter section and the junction “Friedberg”, there are two approximately 10 kilometer long three-lane freeway sections without disturbances by on- or off-ramps. However, the freeway crosses

a valley at kilometer 478 to 480 with gradients of ca. 2-3%, which causes an additional flow-conserving bottleneck in direction North. Moreover, there is a relatively steep hill (“Köpperner Berg”) between kilometers 471 and 472.5 with gradients up to 5%, which often produces congestion patterns in direction South. The main bottlenecks on the considered freeway stretch and their reasons are listed in Table 1, which also contains the estimated bottleneck strengths.

Position	Direction	Reason of Bottleneck	Estimated Strength
km 471.1	South	Junction Friedberg	550 veh./h/lane (max.)
km 481.3	South	Intersection Bad Homburg	320 veh./h/lane
km 488.5	North	Intersection Frankfurt North-West	330 veh./h/lane
km 478.0	North	Gradient	110 veh./h/lane

Table 1: Main bottlenecks on the investigated section of the German freeway A5 north of Frankfurt.

Figure 2: Sketch of the investigated freeway section, showing the two directions of the German three-lane freeway A5 near Frankfurt/Main. Each measurement cross section of the freeway is marked by a vertical line. It is named with the initial of the driving direction (“N” for north or “S” for south), followed by a number increasing in driving direction. The geographical position of each detector is given in kilometers according to the official notation of the responsible road authorities.

We have investigated the data for both driving directions of the above freeway for all the 165 days between 04/01/2001 and 09/30/2001, which allows us to identify the typical features of traffic flow and to make statistical analyses. On these days (distinguishing both directions), we have identified more than 240 breakdowns of traffic flow mostly during the morning rush hour between about 7 am and 9 am and during the afternoon rush hour between about 3 pm and 7 pm (on Fridays between about 1 pm and 7 pm). Breakdowns were also induced by holiday traffic or accidents. The latter normally occur outside of the typical bottleneck areas and are protocolled by the road authorities. During the investigated time period, about 500 accidents have occurred. However, most of them had only a minor impact on the traffic flow, as the cars involved were parked on the emergency lane and did not block any of the freeway lanes.

Our study is based on aggregate double-loop detector data containing, among other information, the arithmetically averaged vehicle velocities and traffic flows at 30 cross



sections (in direction North) or 31 (in direction South), see Fig. 2. We have pre-processed our one-minute data by an error correction which is followed by a smoothing routine. Apart of the few times in a month, when none of the detectors recorded any data for a certain time period due to maintainance work or a breakdown of the computer facilities, the data showed very few errors of two types: (1) Sometimes the data of a cross section just consisted of error bits for a single minute or two subsequent minutes. In this case (amounting to approx. 1% of the data), we applied a linear interpolation in time. If error bits for longer time intervalls than two minutes were found, all values were set to zero. After visualization of the data, these events were clearly visible, because errors of this type occurred simultaneously for all detectors of a direction. (2) In other cases, some detector (measuring a single lane at a specific cross section) failed, which sometimes lasted for a few weeks. When traffic around this cross section was congested, we averaged the data in the other lanes. Because of similar velocities in neighboring lanes during congestion, this procedure leads to realistic velocities. If there was free traffic, we interpolated the data of the same lane at the preceding and the following cross section. Two representative examples of faulty measurements are displayed in Fig. 3. In order

Figure 3: Two Examples of faulty measurements: (a) Between 9:19 and 9:30 there was no data update, and all detectors delivered permanently the data of 9:19. For the time intervalls 9:30 to 9:33 and 9:37 to 9:40 all detectors delivered error bits which were replaced by zero-values. This is the reason for the short episode of low velocities over the whole section after 9:30. The zero-values during this intervalls of 4 minutes cannot be seen clearly because of the applied smoothing (in particular between 9:33 and 9:37 data were available). (b) Example, where the error bits between 10:09 and 10:39 were replaced by zeros.

to get a spatio-temporal impression of the traffic patterns based on the data of a few measurement cross sections, we have applied an adaptive smoothing method, which has shown to deliver realistic results also when the number of measurement cross sections is reduced (Treiber and Helbing, 2002). The adaptive smoothing method uses an exponential filter  $\phi(x, t)$  which smoothes over an average time window  $\tau$  and an average spatial interval  $\sigma$ . In this study, we have always chosen  $\tau = 1.2$  min and  $\sigma = 0.6$  km. The particular feature of the method is the smoothing into the respective propagation direction of perturbations in traffic flow (i.e. along the “characteristic lines”). In free traffic, perturbations are assumed to propagate forward (downstream) with a speed of approximately  $c_{\text{free}} = 80$  kilometers per hour, while in congested traffic, the perturbations travel upstream with about  $c_{\text{cong}} = -15$  kilometers per hour (cf. Fig. 4). These values have been calibrated in a way that minimizes discontinuities in the three-dimensional representation of the average velocity along the freeway in the course of time and agree well with other observations (Kerner and Rehborn, 1996a). The adaptive smoothing method switches automatically between the free and the congested regime based on a certain criterion, so that there is no subjective element in this method of data preprocessing

(for details see Treiber and Helbing, 2002). The main properties and advantages of this method are illustrated by Fig. 5.

We have applied this method to the representation of different quantities such as the average velocity  $V(x, t)$ , its inverse  $1/V(x, t)$ , the traffic flow  $Q(x, t)$ , or the vehicle density determined by  $\rho(x, t) = Q(x, t)/V(x, t)$ . It turned out that the most intuitive picture of the traffic situation is obtained by showing the average velocity over space and time, but with high values on the bottom and small values on the top. In this way, the data evaluation is based on a quantity which is measured in a relatively easy and reliable way (in contrast to the vehicle density), and congestion corresponds to hills, similar to the typical representation of the density. In other words, the higher the value on the vertical axis of the graph, the higher is the resistance to the driver (the smaller is the velocity).

Figure 4: Illustration of the effects of two linear homogeneous filters with the kernels  $\phi^{\text{free}}(x, t)$  and  $\phi^{\text{cong}}(x, t)$ , respectively (from Treiber and Helbing, 2002). The shaded areas denote the regions considered in the calculation of a data point at  $(x, t)$ . Triangles denote the mainly contributing input data sampled in free traffic, squares the ones sampled in congested traffic.

Figure 5: Velocity contour plots of stop-and-go waves observed on 04/05/2001 in direction South. Left: Isotropic smoothing, yielding discontinuous patterns. Right: Same data using the adaptive smoothing method, where travelling waves become clearly visible. (For better illustration, just every second detector has been used here, resulting in an effective mean distance between neighboring detectors of about 2 km. The actual mean distance is approximately 1 km, cf. Fig. 2.) The accident at kilometer 483.1 at 7:20 is clearly visible in the data.

Furthermore, the method smoothes out statistical fluctuations of the measurements (which are due to the fact that one-minute data average over a small number of vehicles only). In this way, the main systematic features of traffic patterns become more easily visible. For an investigation of traffic data from the same freeway stretch with the method of cumulative plots see Bertini *et al.* (2003).

### 3 Empirical Features of Congested Traffic States

In the following section, we will discuss different kinds of congested spatio-temporal traffic patterns which have been observed on the investigated section of the German freeway

A5. The same patterns have been identified on sections of other freeways. Differences in the patterns for other freeways and other countries will be discussed in Sec. 3.9.

### 3.1 Pinned Localized Cluster (PLC)

Let us start with a discussion of the particular kind of spatio-temporal congestion pattern illustrated in Fig. 6. This pattern is characterized by a localized breakdown of velocity and a higher density. Moreover, it has a typical spatial extension. The pattern normally occurs at bottlenecks of the freeway during rush-hours and does not move up- or downstream. As it appears to be pinned to a fixed and well-defined location, it is named a pinned localized cluster (PLC). In some cases, the cluster may oscillate in time, which is called an oscillating pinned localized cluster (OPLC), see Fig. 6b. These oscillating states may be viewed as spatially confined stop-and-go waves (see Sec. 4.2).

Figure 6: Representative examples of pinned localized clusters.

Pinned localized clusters may be formed spontaneously (see Fig. 6a), or they may be caused by an upstream travelling perturbation which stops at the location of the bottleneck (see Fig. 6b). Upstream and downstream of the pinned localized cluster, we have free traffic flow. Pinned localized clusters may, therefore, be caused by slower, entering, or lane-changing vehicles along a bottleneck, e.g. an on-ramp or gradient. When the traffic volume becomes too high, the pinned localized clusters start to expand in space, which gives rise to other, spatially extended congestion patterns.

### 3.2 Homogeneous Congested Traffic (HCT)

One kind of extended congested traffic is homogeneous congested traffic (HCT). Typically, this pattern occurs for heavily congested roads, e.g. after serious accidents or during holiday traffic. In Fig. 7a, for example, we have discovered a complete closing of all three lanes by data analysis, which has later on been confirmed by the responsible road authorities. After a first accident occurred at 13:50 at kilometer 477.08, possibly because of an unexpected, upstream travelling perturbation, 16 other cars were immediately involved in six subsequent accidents. A quarter of an hour later, two more accidents happened at about the same position. The congestion in Fig. 7b was also caused by an accident at 19:15 at kilometer 478.736.

Figure 7: Representative examples of homogeneous congested traffic.

In homogeneous congested traffic, the (smoothed) velocity is very low and more or less constant (i.e. homogeneous) over a longer section of the freeway. The downstream front is located slightly downstream of the upstream end of a serious bottleneck, while the downstream end moves upstream, which gives rise to a spatially extended congestion pattern growing in time. The velocity  $C$  of the upstream shock front appears to be consistent with the Lighthill-Whitham theory, i.e.

$$C(t) = \frac{Q_{\text{cong}}(t) - Q_{\text{up}}(t)}{\rho_{\text{cong}}(t) - \rho_{\text{up}}(t)}, \quad (3)$$

where  $Q_{\text{cong}}(t)$  and  $\rho_{\text{cong}}(t)$  are the flow and density, respectively, inside of the congested area, while  $Q_{\text{up}}$  and  $\rho_{\text{up}}$  are the (free) flow and density in the uncongested area immediately upstream. Downstream of homogeneous congested traffic, one usually finds free traffic. Once the bottleneck is removed (e.g. the accident and the lanes blocked by it are cleared), the downstream front of congested traffic moves upstream with a speed  $c$  which approximately agrees with  $c_{\text{cong}} = -15$  km/h. (A small pinned localized cluster of reduced density may, however, remain at the location of the bottleneck, possibly because of continuing efforts on the emergency lane). The spatial extension of homogeneous congested traffic shrinks as soon as  $|c| > |C|$  (see Fig. 7b). The time of disappearance can, in principle be calculated, when the clearing time and the time-dependent upstream flow  $Q_{\text{up}}(t)$  are known. The upstream density  $\rho_{\text{up}}(t)$  may be calculated from the free branch of the fundamental diagram  $Q(\rho)$  via the formula  $Q_{\text{up}}(t) = Q(\rho_{\text{up}}(t))$ .

### 3.3 Oscillating Congested Traffic (OCT)

Oscillating congested traffic is another kind of extended congestion pattern. It has similar features as homogeneous congested traffic regarding its development, growth and dissolution mechanism. However, the congested area shows more or less regular oscillations of the speed with a frequency and amplitude staying relatively constant over a certain period of time. The oscillations propagate upstream with a velocity  $c$  which approximately agrees with  $c_{\text{cong}} = -15$  km/h.

Figure 8: Representative examples of oscillating congested traffic. The congestion in sub-figure (a) is caused by an accident at kilometer 478.325 at 9:50, while (b) is a result of a possible hindrance on the fast lane between kilometers 486.0 and 486.9.

Oscillating congested traffic is often surrounded by free traffic flow and may be triggered by a perturbation, but it can also arise when the traffic volume exceeds a certain value. Its downstream front is located slightly downstream of the upstream end of a bottleneck, until the bottleneck capacity  $Q_{\text{cap}}$  is sufficient to cope with the on-ramp flow  $Q_{\text{rmp}}(t)$  plus the upstream flow  $Q_{\text{up}}(t)$ . The upstream congestion front propagates again with an

average speed given by Eq. (3), and oscillating congested traffic dissolves in a similar way as homogeneous one (see Sec. 3.2). One may distinguish two cases:

- If the average flow  $Q_{\text{cong}}(t)$  inside of the congested area plus the ramp flow  $Q_{\text{rmp}}(t)$  per lane drop below the capacity  $Q_{\text{cap}}$  of the activated bottleneck, the latter can cope with the overall traffic volume (and becomes inactive). *In this case, the downstream congestion front starts to move upstream* with a speed  $c \approx c_{\text{cong}}$ , until the upstream front is reached and the congestion is thereby dissolved (see Fig. 8a).
- If the overall traffic volume  $Q_{\text{cong}}(t) + Q_{\text{rmp}}(t)$  stays above the bottleneck capacity  $Q_{\text{cap}}$  during the whole congestion period, the upstream congestion front starts to move downstream as soon as the time-dependent upstream flow  $Q_{\text{up}}(t)$  drops below the average congested flow  $Q_{\text{cong}}$  (see Fig. 8b).

### 3.4 Stop-and-Go Waves (SGW)

Another form of congested traffic, which has spatially extended and localized features at the same time, are stop-and-go waves. They are related to oscillating congested traffic, but they have a large characteristic amplitude, while there is no typical wavelength. Stop-and-go waves (SGW) consist of a sequence of traffic jams with free traffic in between. The traffic jams are localized (i.e. spatially confined) and propagate upstream with velocity  $c_{\text{cong}} \approx -15$  km/h. The spatial and temporal distance among two successive traffic jams varies significantly. Stop-and-go waves are sometimes triggered by small perturbations in the traffic flow or may originate from an area of pinned localized clusters (see Figs. 9, 11, and 13). The propagation speed of stop-and-go waves does not change when they travel through pinned localized clusters (see Figs. 11b, 13b) or spatially extended congested traffic (Kerner 2000a, b, 2002). Their propagation typically ends in free traffic. However, they may also end as pinned localized clusters (cf. Fig. 6b).

Figure 9: Representative examples of stop-and-go waves, with an accident in (a) at kilometer 482.8 at 16:20.

### 3.5 Moving Localized Cluster (MLC)

In the case of a single moving traffic jam (instead of sequence of them), we talk about a moving localized cluster (MLC). Compared to a pinned localized cluster, the extension of a MLC state is also limited, but it is propagating with the speed  $c_{\text{cong}}$  rather than staying at a particular place. A moving localized cluster is usually born from a perturbation of traffic flow. One can distinguish two different cases:

- The perturbation is large enough to travel upstream from the very beginning (see Fig. 10a).
- The initial perturbation is small, and one observes a boomerang effect (cf. Fig. 10b and the next subsection).

Figure 10: Representative examples of moving localized clusters. Note that, in the right picture, a small perturbation triggers a pinned localized cluster while the moving localized cluster passes the bottleneck at kilometer 480.

### 3.6 The Boomerang Effect

One typical mechanism for the triggering of the above mentioned congested traffic patterns is the so-called boomerang effect (Helbing, 2001b, Helbing *et al.*, 2003), which can be seen in simulation results of traffic models with a linearly unstable density range (Kerner and Konhäuser, 1993). According to the boomerang effect, small perturbations in free traffic propagate downstream, but if they grow, they change their propagation speed and direction, so that they eventually return. If there is a mechanism which, under certain conditions, causes a growth of small perturbations, such a behavior is expected: Small perturbations correspond to clusters of vehicles, which should move into the direction of the vehicles, i.e. downstream. On the other hand, once a moving traffic jam has developed, it is supposed to move in upstream direction: Inside of the traffic jam, vehicles are standing, at its downstream front, vehicles are leaving, and at the upstream front, new vehicles are joining the standing vehicle queue. Consequently, when a moving traffic jam is born from a perturbation of the traffic flow, the propagation direction should change and eventually turn around.

In Sec. 4.2, we will discuss whether the growth of small perturbations can be explained in a different way, for example based on ramp flows. Here, we just note that, although a breakdown of traffic flow is not always related to the boomerang effect, it occurs rather frequently (see Figs. 6b, 11, and 13b). However, it is hard to recognize without a suitable method of data preprocessing. When no smoothing method is applied, small perturbations are normally masked by the fluctuations. Moreover, if the propagation direction of perturbations is not considered by the interpolation procedure, continuously moving patterns are splitted up into artificial, discontinuously looking structures, and it is hard to make sense of these (see Fig. 5). It is also hard to identify the boomerang effect without a three-dimensional representation, just on the basis of discrete detector data. Therefore, the empirical identification of the boomerang effect has profited from advanced methods of data preprocessing such as the adaptive smoothing method discussed in Sec. 2. It is highly unlikely that this effect is an artefact of the data representation, as it can sometimes also be seen in classical contour plots without particular preprocessing (Helbing *et al.*, 2003) and in three-dimensional plots without a spatial smoothing (see Fig. 12).

Figure 11: Representative examples of the boomerang effect (see the area around kilometer 485 and Figs. 6b, 13b).

Figure 12: Illustration of growing perturbations on a homogeneous freeway section without on- and off-ramps. Growth of a small, downstream travelling perturbation and boomerang effect represented (a) by the adaptive smoothing method and (b) by time-dependent velocity data obtained at the different measurement cross sections (without any spatial smoothing), as seen from behind. (c), (d) Examples for the growth of medium-sized perturbations propagating upstream.

### 3.7 Combined States and the “Pinch Effect”

Freeways are not at all spatially homogeneous. Due to the existence of on- and off-ramps, gradients, curves etc., a freeway can be imagined to be composed of road sections of different capacities, even if the number of lanes is constant. If the capacity is reduced from one road section to the next one in downstream direction, we speak of a bottleneck. Upstream of bottlenecks, there is a danger of queue formation or congestion, if the traffic volume becomes too high, while downstream of the upstream end of a bottleneck, one mostly observes free traffic, if there is not another bottleneck downstream. Congestion normally starts to form upstream of the freeway section, for which the difference between the arriving traffic volume and the section capacity (bottleneck capacity) is highest. With increasing traffic volume, more and more bottlenecks are activated. Therefore, we normally have a sequence of different congested traffic patterns along the freeway.

Figure 13: Combinations of different congested traffic states.

For example, Figure 11a shows the formation of a pinned localized cluster upstream of kilometer 471.5 at about 6:30 am. About 30 minutes later, this state turns into stop-and-go traffic, and a weaker bottleneck at kilometer 474 is sometimes activated as well. Around 7:50 am, we observe the formation of a moving localized cluster, which is triggered by the boomerang effect. The moving localized cluster turns into a pinned localized cluster at kilometer 482, which later on triggers pinned localized clusters and stop-and-go waves at kilometer 479 (possibly because of an accident at kilometer 479.2 at 8:50) and at kilometer 469.

In Figure 11b, the boomerang effect triggers a moving localized cluster at about 7:00am. Around 7:30am, a pinned localized cluster is formed at kilometer 480, which also appears to be triggered by a small perturbation which moves downstream and hits the moving

localized cluster around kilometer 480. The moving localized cluster continues, while the pinned localized cluster emits several traffic jams, so that we face triggered stop-and-go waves. Some of the traffic jams propagate through the pinned localized cluster which has formed around 7:30am at kilometer 470 and also generates stop-and-go waves. After 9:00am, some of the stop-and-go waves and the pinned localized traffic at kilometer 470 start to disappear, while the pinned localized cluster at kilometer 480 is still alive at 9:40am.

Figure 13a shows stop-and-go waves upstream of kilometer 480. Around 17:20, homogeneous congested traffic forms at kilometer 488, and the downstream flow is considerably reduced. This is a result of an accident at kilometer 487.5 at 17:13. One hour later, the accident has been cleared. This turns the homogeneous congested traffic into less severe oscillating congested traffic. Moreover, the downstream flow is back to normal, which causes a queue upstream of the bottleneck at kilometer 480. The congestion pattern emerging over there appears to be oscillating congested traffic as well.

Figure 13b illustrates the formation of a moving localized cluster via a boomerang effect around 7am. While the cluster passes along the bottleneck at kilometer 478, it meets a downstream moving perturbation, which triggers the formation of a pinned localized cluster. Around 8am, the moving localized cluster arrives at the bottleneck at kilometer 471.5 where it triggers oscillating congested traffic that persists until 9 o'clock. Around 8:30 am, the pinned localized cluster at kilometer 482 turns into extended congested traffic, and the resulting pattern reminds of a pinch effect.

Figure 14 may be viewed as an illustration of the so-called pinch effect. After 7:00am, we find something like homogeneous congested traffic or a pinned localized cluster at kilometer 480, which turns into oscillating congested traffic further upstream. Some of these oscillations disappear as they travel upstream, while two of them form stop-and-go waves. These enter another area of congested traffic upstream of kilometer 471.5. Note that around kilometer 478, we do not really see a merging of small oscillations in favour of a few remaining moving traffic jams, in contrast to the suggestion by Kerner (1998a). The oscillations rather seem to disappear, i.e. they seem to be dissolved in free traffic. Moreover, the pinch effect was not frequently observed by us. However, both of this may be due to the applied smoothing procedure. Structures on a smaller scale than  $\sigma = 0.6$  km may actually merge, but from our point of view, it is hard to tell them apart from fluctuations.

Figure 14: Representative example of the “pinch effect”, i.e. a spatial sequence of homogeneous congested, oscillating congested, and stop-and-go traffic in upstream direction.



### 3.8 Comparison with Kerner’s Observations

Comparing our empirical results with Kerner’s findings for the same freeway section, we suggest the following identifications: Homogeneous congested traffic (HCT) seems to be the same as “synchronized” traffic flow (ST) of type (i) (see Sec. 1.4), while oscillating congested traffic (OCT) seems to relate to synchronized flow of type (iii). Finally, homogeneous-in-speed states remind of the free branch of the fundamental diagram, but with a reduced free velocity. Therefore, this synchronized flow of type (ii) bears features of both, free and congested traffic. This may point to recovering traffic downstream of bottlenecks (see Fig. 4 in Treiber and Helbing, 1999, and Fig. 3 in Tilch and Helbing, 2000) or to frustrated drivers, who have decreased their desired speed after having spent a considerable amount of time in congested traffic (see Treiber and Helbing, 2003).

### 3.9 Occurrence Frequency of Traffic States and Relevance for Other Sites

We have also investigated the occurrence frequency of the above mentioned traffic states on the investigated freeway stretch. The result is shown in Figs. 15 and 16. According to

Figure 15: Absolute frequencies of congested traffic states on the German freeway A5 close to Frankfurt in direction North.

Figure 16: Absolute frequencies of congested traffic states in direction South.

this, oscillating congested traffic and pinned localized clusters are very common on the studied freeway stretch. Homogeneous congested traffic is less frequent than oscillating congested traffic, in accordance with expectations. If it occurred frequently, the freeway capacity would not be appropriately dimensioned to cope with the traffic volume. We should, however, note that pinned localized clusters cannot always be clearly distinguished from homogeneous congested traffic, if the upstream extension of the latter is limited by a section of high capacity or an off-ramp, where many vehicles (can) leave. This is, for example, potentially the case around kilometer 481. In some particular cases, it is also difficult to exactly tell apart moving localized clusters from stop-and-go waves, or the latter from oscillating congested traffic. Nevertheless, it is helpful and makes sense to distinguish the five congestion patterns mentioned above.

We should mention that one can notice a typical dependence of the frequency of congestion on the day of the week (see Figs. 15 and 16). On Saturdays and Sundays, there is little congestion compared to working days, as expected. Moreover, in direction North, congestion patterns appeared seldomly on Mondays compared to Tuesdays, Wednesdays, or Thursdays (Helbing *et al.*, 2003), while in direction South, the frequency of

traffic breakdowns was surprisingly low on Fridays. This is probably due to commuters spending their weekends in the North and living in Frankfurt during the week.

It should be mentioned that only less than five percent of congestion patterns remained unexplained. The remaining ones were probably caused by accidents, or they were cases of forwardly moving “phantom” bottlenecks (Gazis and Herman, 1992; Muñoz and Daganzo, 2002b), which were found two times (see Fig. 17). Although we could not anymore

Figure 17: Congestion pinned at moving bottlenecks, the velocity is about 1.6 km/h (left) and quite exactly 2.7 km/h (right).

identify the exact reason for the forwardly moving congestion patterns, it is plausible to assume an exceptionally slow heavy goods or road work vehicle (for example, cutting grass along the freeway). In summary, it can be said that the great majority of congestion patterns on the studied freeway section was related to bottlenecks, but some of them were triggered by small perturbations, e.g. via the boomerang effect (in 18 out of 245 cases).

Let us now shortly address the question, whether the above findings are also relevant for other freeway sites and other countries. Very similar congestion patterns have been found on other German freeways (e.g. A3, A8, A9) or on freeways in the Netherlands (e.g. the freeways A2, A9). However, the relative frequency of the different congestion patterns is site-dependent. It seems to depend on the bottleneck strengths and the respective traffic volumes. Moreover, although there are a few reports of oscillating features of congestion in the United States (Daganzo *et al.*, 1999; Cassidy and Bertini, 1999), it appears that oscillating congested traffic and stop-and-go waves occur less frequent than, for example, in Germany. This shows that the frequency of congestion patterns is also a matter of country and driver behavior, probably because parameters such as the average time gap between vehicles, the acceleration behavior of drivers, the lane changing frequency and traffic regulations matter, as well as speed limits and the velocity variance. In any case, we are not aware of additional congestion patterns in the US, which have not been covered by the above classification. In Japan, by the way, oscillating congestion patterns (OCT, SGW) seem to exist (Koshi *et al.*, 1983).

We should finally note that the observations also depend to a certain extent on the detection technology. For example, pinned localized clusters may be overlooked, if detectors are not placed at suitable locations. Moreover, bottlenecks can have various origins: Apart from classical bottlenecks such as on-ramps, gradients and accidents, there are bottlenecks due to off-ramps (caused by frequent lane changes, weaving flows), curves, bad weather, changes in illumination (tunnel entrances, blinding sun), or circumstances that reduce the attention of the drivers to road traffic (beautiful views, accidents on the opposite lanes), etc.

## 4 Comparison with Traffic Models

The identification of empirical traffic patterns is not only of importance to increase the comfort, efficiency and safety of freeway traffic by technological means, e.g. driver assistance systems. It is also helpful to verify traffic models used to develop and assess these measures, to identify the traffic state between measurement sections, and to forecast traffic. We will, therefore, discuss in the following, to what extent the above observations are consistent with first- and second-order macroscopic traffic models and their microscopic equivalents (e.g. car-following models).

### 4.1 The Lighthill-Whitham-Richard Model (LWR Model)

Let us begin with the fluiddynamic model by Lighthill and Whitham (1955) and Richard (1956). It is based on the continuity equation

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial Q(\rho(x, t))}{\partial x} = \nu_+(x, t) - \nu_-(x, t), \quad (4)$$

where  $\rho(x, t)$  denotes the vehicle density as a function of location  $x$  and time  $t$ . For the time being, we will set the source terms  $\nu_{\pm}(x, t)$  due to ramp flows equal to 0 (where the plus sign corresponds to on-ramps and the minus sign to off-ramps). The model is closed by assuming a flow-density relationship  $Q(\rho)$ , which is called the fundamental diagram. The resulting fluid-dynamic traffic model assumes an instantaneous adaptation of the average speed to the density, as it does not contain an independent partial differential equation for the dynamics of the average velocity. Therefore, it is called a first-order model.

We can rewrite Eq. (4) as a non-linear wave equation

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{dQ(\rho)}{d\rho} \frac{\partial \rho(x, t)}{\partial x} = 0. \quad (5)$$

According to this, the density profile propagates with the speed

$$c(\rho) = \frac{dQ(\rho)}{d\rho}, \quad (6)$$

i.e. a formal solution of Eq. (5) is given by

$$\rho(x, t) = \rho\left(x - \int_0^t dt' c\left(\rho(y_x(t'), t')\right), 0\right), \quad (7)$$

where

$$y_x(t') = x - \int_{t'}^t dt'' v(t'') \quad \text{with} \quad \frac{dy_x(t')}{dt'} = v(t') = c\left(\rho(y_x(t'), t')\right) \quad (8)$$

is the location with vehicle density  $\rho$  as a function of time  $t'$ . As this defines an implicit equation for the density  $\rho(x, t)$ , it is hard to solve explicitly. However, we can see from it that the initial density profile  $\rho(x, 0)$  changes its shape in the course of time if the propagation velocity  $c(\rho)$  is density-dependent, but it does not change its amplitude. This is the reason for the evolution of shock fronts, which propagate with the velocity specified in Eq. (6) (Whitham, 1974).

The first-order model (4) is suitable to describe the propagation of the upstream shock fronts of spatially extended congestion patterns such as homogeneous or oscillating congested traffic. It may also describe the propagation of *fully developed* traffic jams, both moving localized clusters and stop-and-go waves (Helbing, 2003). However, it cannot describe the *emergence* of these patterns and of oscillating congested traffic without particular assumptions. Such kinds of assumptions have been formulated. For example, Cassidy and Mauch (2001) and Daganzo (2002b) suggest that these patterns are produced by ramp flows. In particular, they propose a pumping effect at ramps which may explain structures of increasing amplitude. Such an assumption makes sense for freeways in the United States, where on- and off-ramps often have very short distances from each other. In Europe, however, there are freeway sections of about 10 kilometer length without any ramps or changes in the number of lanes. On these sections, according to the above theory, we should not find patterns of increasing amplitude (where we define the amplitude as the difference between the minimum and maximum value of the pattern). This does, by the way, not only apply to the vehicle density  $\rho(x, t)$ , but also to the average vehicle velocity  $V(x, t) = Q(\rho(x, t))/\rho(x, t)$ . As  $V(\rho) = Q(\rho)/\rho$  is a monotonically falling function of the density  $\rho(x, t)$ , minima of the vehicle density correspond to maxima of the average velocity and vice versa.

The boomerang effect questions the first-order model (4), as it shows that patterns of growing amplitude exist even on freeway sections without ramps. Figure 12 illustrates this in more detail by comparing the profiles at subsequent measurement cross sections without any spatial interpolation. As a conclusion, our empirical results call for an extension of the Lighthill-Whitham theory. This should be able to describe patterns with growing amplitudes and to explain emergent oscillating structures without having to assume oscillations in the ramp flows triggering them. Figure 18 shows that the oscillation frequency of triggered oscillating congested traffic or stop-and-go waves is not at all comparable with the frequency of the variations in the ramp flow.

## 4.2 Phase Diagram of Traffic States for Second-Order Models

Compared to the Lighthill-Whitham-Richard model, second-order macroscopic traffic models do not assume an immediate adaptation of the vehicle velocity to a changing traffic situation. They contain an additional partial differential equation for the spatio-temporal change of the average velocity  $V(x, t)$  of vehicles, which can often be written

Figure 18: Oscillating congested traffic on 07/16/2001 at the Junction Friedberg. The oscillation frequency of the on-ramp flow at kilometer 470.8 (upper left) is much higher than the frequency of the flow approximately 2.4 kilometers upstream (upper right). This visual impression is confirmed by the corresponding Fourier spectra (lower figures). The on-ramp flow has a typical frequency larger than 0.3 per minute, while the upstream flow shows a typical frequency of less than 0.1 per minute. Therefore, the frequency of oscillating congested traffic is not determined by the variations in the on-ramp flow.

in the form

$$\frac{\partial V}{\partial t} + \underbrace{V \frac{\partial V}{\partial x}}_{\text{Transport Term}} = \underbrace{-\frac{1}{\rho} \frac{\partial P}{\partial x}}_{\text{Pressure Term}} + \underbrace{\frac{1}{\tau}(V_e - V)}_{\text{Relaxation Term}}. \quad (9)$$

Herein,  $P$  is called the traffic pressure,  $\tau$  the relaxation or adaptation time  $\tau$ , and  $V_e$  the “optimal” (or dynamic equilibrium) velocity  $V_e$ , which is dependent on the local vehicle density  $\rho$  and possibly on other variables as well. Notice that the Lighthill-Whitham model results in the limit  $\tau \rightarrow 0$ , but Whitham has proposed a second-order extension himself (1974). The models by Payne (1971) and Papageorgiou (1983) are obtained for  $P(\rho) = [V_0 - V_e(\rho)]/(2\tau)$ , with the “free” or “desired” average velocity  $V_0 = V_e(0)$ . For  $dP/d\rho = -\rho/[2\tau(\rho + \kappa)]dV_e/d\rho$ , one ends up with Cremer’s (1979) model. In the model of Phillips (1979), there is  $P = \rho\theta$ , where  $\theta$  denotes the velocity variance. The model of Kühne (1984, 1987) and of Kerner and Konhäuser (1993) results for  $P = \rho\theta_0 - \eta\partial V/\partial x$ , where  $\theta_0$  is a positive constant and  $\eta$  a viscosity coefficient. In comparison with a similar model by Whitham (1974), the additional contribution  $-\eta\partial V/\partial x$  implies a viscosity term  $(\eta/\rho)\partial^2 V/\partial x^2$ . This is essential for smoothing shock fronts, which is desirable from an empirical and numerical point of view.

Second-order models have been seriously criticized by Daganzo (1995c), but his criticism has been overcome by improved macroscopic traffic models (Helbing, 1995a, b, c, 1996, 1997a, 2001a; Aw and Rascle, 2000; Helbing *et al.*, 2001; Aw *et al.*, 2002). The non-local, gas-kinetic-based traffic (GKT) model, for example, is theoretically consistent, numerically efficient, realistic, and has been successfully applied for traffic state identification in large freeway networks. Instead of introducing a smoothing effect via a viscosity term, it considers the reaction of drivers to the traffic situation ahead of them (i.e. anticipation effects). This basically causes an additional dependence of the “optimal velocity” from the density and average velocity at an advanced location, which makes the model non-local. It also takes into account effects of space requirements by vehicle lengths and safe time clearances. The traffic pressure is not only density-dependent, but proportional to the squared average velocity, which avoids the possibility of negative velocities. Correlations between the velocities of successive cars can be easily treated as well. Here, we will not repeat the detailed mathematical form and properties of this model, as they have been described in several references (Helbing and Treiber, 1999; Helbing, 2001; Helbing *et al.*,

2001, 2002). Moreover, qualitatively the same traffic states and preconditions for their occurrence have been found for other second-order models such as the Kerner-Konhäuser model (Lee *et al.*, 1999). We will only summarize the main points shortly in this paper, as detailed discussions are available elsewhere (Helbing and Treiber, 1999; Helbing, 2001; Helbing *et al.*, 2001, 2002).

The gas-kinetic-based traffic model produces free traffic (FT) and five different kinds of congested traffic states, which are displayed in Fig. 19. The typical preconditions for

Figure 19: Simulation of a freeway with an on-ramp at location  $x = 5$  km and an initial disturbance travelling upstream, using the non-local, gas-kinetic-based traffic (GKT) model (Helbing *et al.*, 1999). Five different kinds of congested traffic states may emerge, depending on the respective traffic flows on the ramp and on the freeway: a moving localized cluster (MLC), a pinned localized cluster (PLC), stop-and-go waves (SGW), oscillating congested traffic (OCT), and homogeneous congested traffic (HCT). During the first minutes of the simulation, the flows on the freeway and the on-ramp were increased from low values to their final values.

their occurrence can be illustrated by a phase diagram (see Fig. 20). Each area of the phase diagram represents the parameter combinations, for which certain kinds of traffic state can exist. It is interesting that the borderlines between different areas (the so-called phase boundaries) can be theoretically understood based on the instability diagram and the dynamic capacity  $Q_{\text{out}}$ . The latter is given by the outflow from congested traffic and has typical values of  $1800 \pm 100$  vehicles per hour and lane (Kerner and Rehborn, 1996a).

Figure 20: Theoretical phase diagram of traffic states with numerical estimates of the boundaries for the bottleneck around kilometer 470 on the German freeway A5 in direction South. The different areas indicate, for which combinations of the upstream freeway flow  $Q_{\text{up}}$  and the bottleneck strength  $\Delta Q$  certain traffic states may exist, depending on the initial and boundary conditions. Most areas are multistable, i.e. one may find one out of several possible states.

The non-local, gas-kinetic-based traffic (GKT) model and some other second-order traffic models predict stable traffic flow, when the velocity changes little with the density. More specifically, there is stable traffic below some critical density  $\rho_{c1}$  and above some critical density  $\rho_{c4}$  (see Fig. 21).

Figure 21: Schematic illustration of velocity  $V$  and flow  $Q$  as a function of the vehicle density  $\rho$ . Grey regions indicate density ranges of metastable traffic (cf. text).

For medium traffic densities between the critical densities  $\rho_{c2}$  and  $\rho_{c3}$ , traffic flow is linearly unstable, i.e. even the smallest perturbation can grow and cause a breakdown of traffic flow. In the intermediate density ranges  $\rho_{c1} \leq \rho < \rho_{c2}$  and  $\rho_{c3} < \rho \leq \rho_{c4}$ , one finds metastable traffic, i.e. sufficiently small perturbations will fade away, while large enough ones will grow and cause a breakdown of traffic flow. The value of  $Q_{\text{out}}$  falls into the metastable regime between  $\rho_{c1}$  and  $\rho_{c2}$ .

Let us now investigate a bottleneck situation due to inflows

$$\nu_+ = \frac{Q_{\text{rmp}}}{nL} \quad (10)$$

along a ramp, where  $L$  is the used length of the on-ramp and  $n$  the number of freeway lanes. The corresponding bottleneck strength is, then,  $\Delta Q = Q_{\text{rmp}}/n$ . Moreover, let  $Q_{\text{up}}$  denote the traffic flow upstream of the bottleneck and

$$Q_{\text{tot}} = Q_{\text{up}} + \Delta Q \quad (11)$$

the total capacity required downstream of it. Then, we expect to always observe free traffic (FT) below the threshold  $Q(\rho_{c1})$  of metastable traffic, i.e. for

$$Q_{\text{tot}} = Q_{\text{up}} + \Delta Q < Q(\rho_{c1}) . \quad (12)$$

In contrast, traffic flow will always be congested, if the maximum flow  $Q_{\text{max}} = \max_{\rho} Q(\rho)$  (the capacity) is exceeded, i.e.

$$Q_{\text{tot}} = Q_{\text{up}} + \Delta Q > Q_{\text{max}} . \quad (13)$$

Assuming model parameter for which the maximum flow  $Q_{\text{max}}$  lies between  $\rho_{c1}$  and  $\rho_{c2}$ , traffic states between the two diagonal lines  $Q_{\text{up}} + \Delta Q = Q(\rho_{c1})$  and  $Q_{\text{up}} + \Delta Q = Q_{\text{max}}$  in the  $\Delta Q$ -over- $Q_{\text{up}}$  phase space can be either congested or free, depending on the initial and boundary conditions. While homogeneous free flow may persist over long time periods, large perturbations tend to produce congested states. Extended congested traffic can emerge above the line

$$Q_{\text{up}} = Q_{\text{out}} - \Delta Q \quad (14)$$

in the phase diagram, i.e. if required capacity  $Q_{\text{tot}}$  is greater than the dynamic capacity  $Q_{\text{out}}$ . This line does not have to be parallel to the previously mentioned phase boundaries, as  $Q_{\text{out}}$  may depend on the bottleneck strength  $\Delta Q$ , but it depends somewhat on the bottleneck strength  $\Delta Q$  (Treiber, Hennecke, and Helbing, 2000; Helbing, 2001a). For  $Q(\rho_{c1}) \leq Q_{\text{tot}} = Q_{\text{up}} + \Delta Q < Q_{\text{out}}$ , congested traffic states are expected to be localized, i.e. they should not extended over long freeway sections.

The traffic flow  $Q_{\text{cong}}$  resulting in the congested area plus the inflow or bottleneck strength  $\Delta Q$  are normally given by the outflow  $Q_{\text{out}}$ , i.e.

$$Q_{\text{cong}} = Q_{\text{out}} - \Delta Q \quad (15)$$

(if vehicles cannot enter the freeway downstream of the congestion front). One can distinguish the following cases: *Homogeneous congested traffic* (HCT) can occur, if the density  $\rho_{\text{cong}}$  associated with the congested flow

$$Q_{\text{cong}} = Q(\rho_{\text{cong}}) \quad (16)$$

lies in the stable or meta-stable range

$$Q_{\text{cong}} \leq Q(\rho_{c3}), \quad \text{i.e.} \quad \Delta Q \geq Q_{\text{out}} - Q(\rho_{c3}). \quad (17)$$

Oscillating forms of congested traffic can emerge, if

$$\Delta Q \leq Q_{\text{out}} - Q(\rho_{c4}) \quad \text{and} \quad Q_{\text{up}} > Q_{\text{out}} - \Delta Q. \quad (18)$$

That is, lower bottleneck strengths tend to produce oscillating rather than homogeneous congested flow, and we find *oscillating congested traffic* (OCT), *stop-and-go waves* (SGW), or *moving localized clusters* (MLC). In contrast to OCT, stop-and-go waves are characterized by a sequence of moving jams, between which traffic flows freely. Under certain conditions, each traffic jam can trigger another one by inducing a small perturbation in the inhomogeneous freeway section, which propagates downstream as long as it is small, but turns back when it has grown large enough (*“boomerang effect”*). This requires the downstream traffic flow to be linearly unstable or at least meta-stable. If it is metastable, however, (when the traffic volume  $Q_{\text{tot}}$  is sufficiently small), i.e. if

$$Q(\rho_{c1}) \leq Q_{\text{tot}} = Q_{\text{up}} + \Delta Q < Q(\rho_{c2}), \quad (19)$$

small perturbations may fade away. In that case, one either finds a single *moving localized cluster* (MLC), or a *pinned localized cluster* (PLC) at the location of the ramp. The latter requires the traffic flow in the upstream section to be stable, i.e.

$$Q_{\text{up}} < Q(\rho_{c1}), \quad (20)$$

so that no traffic jam can survive there. In contrast, moving localized clusters and triggered stop-and-go waves require

$$Q_{\text{up}} \geq Q(\rho_{c1}). \quad (21)$$

In practical situations, things are a little bit more complicated:

- The phase diagram depicted in Fig. 20 assumes a single bottleneck only, while in many cases one is, for example, confronted with a combination of off- and on-ramps. Therefore, if traffic upstream of an on-ramp is congested, drivers may react to this by leaving the freeway over the off-ramp. This behavior can suppress a growth of congested traffic beyond the off-ramp. As a consequence, the upstream front of extended congested traffic may be pinned, when it reaches the location of an off-ramp, and OCT states may look like OPLC states, while HCT states may sometimes look like PLC states.



- More detailed simulation studies show that there are also multistable areas, where either PLC, OCT, or FT states can emerge, depending on the respective initial condition. This is particularly the case, if the outflow  $Q_{\text{out}}$  from congested traffic does not exactly agree with  $Q(\rho_{c2})$  (Lee, Lee and Kim, 1999; Treiber, Hennecke, and Helbing, 2000).
- For particular conditions, one also finds traffic states reminding of the “pinch effect”, which is characterized by a spatial coexistence of HCT, OCT, and SGW states (Helbing, 2001a). More specifically, the HCT state is found upstream of a bottleneck and turns into an OCT state further upstream (the “pinch region”). As the small oscillations travel upstream, they merge to form a SGW state. The empirically measured relation between the wavelength of the oscillations and the maximum vehicle velocity (Kerner, 1998a) is convincingly reproduced by simulations (Helbing *et al.*, 2003).
- In principle, phase diagrams can also be generated for more complex freeway geometries with several bottlenecks. These are, however, not two- but multidimensional (with one additional dimension per bottleneck). For this reason, there will be many more phases, corresponding to the possible combinations of the six traffic states FT, PLC, MLC, SGW, OCT, and HCT at different bottlenecks and cases where extended traffic states influence the traffic states at bottlenecks further upstream. All these complex states can be simulated on a computer, when the location-dependent capacities of the freeway plus the boundary flows and ramp flows are known.
- In cases of long ramps, vehicles may enter the freeway downstream of the congestion front induced by it. The downstream flow per lane may, therefore, be higher than  $Q_{\text{out}}$ .
- The large scattering of traffic flows due to variations in the netto time gaps (see Banks, 1999; Treiber and Helbing, 1999; Nishinari *et al.*, 2003) makes it difficult to locate the exact point of empirical traffic states in the phase diagram. Therefore, the empirical phase boundaries are expected to have a fuzzy appearance and to depend on the respective truck fraction.

Despite of the above complications, the empirical phase diagram depicted in Fig. 22 is surprisingly well compatible with the theoretical one (see Fig. 20):

- For values of the overall flow  $Q_{\text{tot}} = Q_{\text{up}} + Q_{\text{rmp}}/(nL)$  below 1300 vehicles per hour and lane, traffic is always free, while above 2100 vehicles per hour and lane, it is always congested.
- Moving localized clusters occur for upstream flows above 1300 vehicles per hour and lane and for overall flows  $Q_{\text{tot}}$  below 1800 vehicles per hour and lane.

Figure 22: Phase points of observed traffic states in the phase space spanned by the effective on-ramp flow  $Q_{\text{rmp}}/n$  per freeway lane and the upstream freeway flow  $Q_{\text{up}}$  per lane (F = free traffic, M = moving localized cluster, S = stop-and-go waves, O = oscillating congested traffic, L = localized traffic [probably not pinned localized clusters, but oscillating or homogeneous congested traffic that did not pass the off-ramp]). The displayed flow values are averages over the 10 minutes immediately before the breakdown of traffic flow. Solid lines are supposed to be guides to the eyes to support a comparison with the theoretical phase diagram displayed in Fig. 20.

- For  $1800 \text{ veh./h/lane} \leq Q_{\text{tot}} \leq 2100 \text{ veh./h/lane}$  and  $Q_{\text{up}} \geq 1300 \text{ veh./h/lane}$ , we find a coexistence of free traffic, oscillating congested traffic, and stop-and-go waves.
- Homogeneous congested traffic is only found after serious accidents with lane closures, with a bottleneck strength  $\Delta Q \geq 900$  vehicles per hour and lane (not shown).

Altogether, there is a good agreement between the theoretical and empirical traffic states, and the empirical phase diagram is qualitatively compatible with the theoretical phase diagram. The only debatable point is the interpretation of the localized states (L) as extended congested traffic states that did not pass the off-ramp due to drivers leaving the freeway. We will, therefore, discuss another, flow-conserving bottleneck caused by a gradient (see Fig. 23). In this case, stationary localized clusters must be actually pinned localized clusters (PLC), but the bottleneck strength  $\Delta Q$  is hard to measure. For this reason, we do not have a two-dimensional phase diagram, but an intersection through it. Assuming that the bottleneck strength can be estimated by  $\Delta Q \approx C_1 Q_{\text{up}}$ , the phase diagram predicts the following sequence of traffic states with growing freeway flows  $Q_{\text{up}}$ : free traffic  $\longrightarrow$  pinned localized clusters or free traffic  $\longrightarrow$  moving localized clusters or free traffic  $\longrightarrow$  oscillating congested traffic, stop-and-go waves, moving localized clusters, or free traffic  $\longrightarrow$  oscillating congested traffic or stop-and-go waves  $\longrightarrow$  oscillating congested traffic or homogeneous congested traffic  $\longrightarrow$  homogeneous congested traffic. Despite the wide scattering of traffic flow data, this theoretically predicted sequence is compatible with our empirical observations (see Fig. 20). The empirical data suggest the following values for the gradient bottleneck:  $Q(\rho_{\text{cl}}) \approx 1700 \text{ veh./h/lane}$ ,  $Q_{\text{out}} \approx 1900 \text{ veh./h/lane}$ , and  $C_1 \approx 0.06$ .

### 4.3 Relevance for Other Traffic Models

Is the non-local, gas-kinetic-based traffic model (GKT model) the only one that produces qualitatively correct traffic states and phase diagrams? The answer is no. According to

Figure 23: Empirical traffic states at the gradient around kilometer 479 on the German freeway A5 in direction North. Despite the considerable scattering, one observes a tendency that pinned localized clusters (P) occur at higher upstream flows  $Q_{\text{up}}$  than free traffic (F), and even higher flows tend to produce moving localized clusters (M), stop-and-go waves (S), or oscillating congested traffic (O).

the above theoretical explanation of the phase boundaries, the same properties are expected for all traffic models with a similar instability diagram. It is particularly essential to have a density regime  $\rho_{c1} \leq \rho < \rho_{c2}$ , in which traffic is metastable. Moreover, the outflow  $Q_{\text{out}}$  from congested traffic needs to be smaller than the maximum homogeneous traffic flow. Finally, there should be a critical density  $\rho_{c3}$ , above which traffic is metastable or stable. The GKT model has these features in common with, for example, the Kerner-Konhäuser model (1994), the optimal velocity model (Bando *et al.*, 1995b), and the intelligent driver model (Treiber *et al.*, 2000). It is, therefore, not surprising that qualitatively the same traffic states and phase diagrams have been found for them. The reader can check this by means of the Java simulation applet of an on-ramp scenario supplied at <http://www.mtreiber.de>. By variation of the main flow and the on-ramp flow, it is possible to produce different kinds of traffic states.

This suggests that phase diagrams of traffic states may be used to classify traffic models, no matter whether they are microscopic, cellular automata, car-following, gas-kinetic or macroscopic models (see Sec. 1). According to the above, the IDM model, which is a microscopic car-following model, may be called equivalent to the macroscopic Kerner-Konhäuser model or the GKT model, when it comes to qualitative features of traffic states. Other models have different instability and phase diagrams:

- Models without a metastable density regime  $\rho_{c1} \leq \rho < \rho_{c2}$  will not show the localized MLC and PLC states, although they may still possess SGW states. According to Krauß (1998), traffic models show the observed metastable traffic and a capacity drop only, if the typical maximal acceleration is not too large and if the deceleration strength is moderate.
- Models without a stable high-density range will not show a HCT state. For example, if the model displays stable traffic at small densities and unstable traffic at high densities, one expects free traffic (FT) and oscillating traffic (OCT or SGW). This was observed by Emmerich and Rank (1995) (see also Diedrich *et al.*, 2000; Cheybani *et al.*, 2001).
- Models without a linearly unstable regime (such as the Burgers equation or the LWR model) will not produce emergent oscillating states (OCT or SGW). For the totally asymmetric exclusion process (TASEP), a simple microscopic particle

hopping model, this has been shown to be in agreement with numerical (Janowsky and Lebowitz, 1992) and exact analytical results (Schütz, 1993).

Although these model classes have different phase diagrams, all of them produce extended congested traffic states (vehicle queues) at bottlenecks.

## 5 Summary and Discussion

In this contribution, we have presented results of a systematic analysis of empirical traffic states of a 30 kilometer long section of the German freeway A5 near Frankfurt. Despite two freeway intersections, there are two approximately 10 kilometer long freeway sections without any on- or off-ramps. Along the freeway section, one finds a rich variety of congested traffic states, but the great majority of them can be interpreted as a spatial coexistence of altogether six different traffic states: free traffic (FT), pinned localized clusters (PLC), moving localized clusters (MLC), stop-and-go waves (SGW), oscillating congested traffic (OCT), and homogeneous congested traffic (HCT). Note that the existence of homogeneous congested traffic points to stable traffic flow at very high densities or very low velocities.

The typical traffic pattern depends on the overall flow (i.e. the upstream freeway flow plus the ramp flow) and, therefore, on the day of the week, but due to multi-stability, initial and boundary conditions are relevant as well. The most frequent states at the investigated freeway section are PLC and OCT states, while HCT occurs mainly after serious accidents with lane closures or during public holidays. The downstream fronts of these congestion patterns are located at bottlenecks.

Note that bottlenecks may have different origins: on-ramps, reductions in the number of lanes, accidents (even in opposite lanes because of rubbernecks), speed limits, road works, gradients, curves, bad road conditions (possibly due to rain, fog, or ice), bad visibility (e.g., because of blinding sun or tunnel entrances), diverges (“negative” perturbations, see Helbing *et al.*, 2003), weaving flows by vehicles trying to switch to the slow exit lane, or congestion on the off-ramp, see Daganzo *et al.*, 1999; Lawson *et al.*, 1999; Muñoz and Daganzo, 1999; Daganzo, 2002a). Moving bottlenecks due to slow vehicles are possible as well (Gazis and Herman, 1992), leading to a forward movement of the respective downstream congestion front (see Fig. 17). Finally, in cases of *two* subsequent inhomogeneities of the road, there are forms of congested traffic in which both, the upstream and downstream fronts are locally fixed (Treiber *et al.*, 2000; Lee *et al.*, 2000; Kerner, 2000a).

For the identification of the traffic states, we have used an adaptive smoothing method which interpolates and smoothes traffic data from successive freeway sections, taking into account the propagation speeds of perturbations in free and congested traffic. The

method is based on measurements of average vehicle speeds by double-loop detectors, which are quite reliable. Our inverse speed representation reminds of density plots, which is particularly intuitive. It also allows one to identify small structures such as the boomerang effect, which is one of the mechanisms of congestion formation: Perturbations in free traffic travel downstream as long as they are small. However, in some cases they grow in amplitude and change their speed and direction of motion, similar to a boomerang. When the considerably amplified perturbation reaches a bottleneck, it can cause a breakdown of traffic flow, as the outflow from congested traffic is smaller than the maximum homogeneous flow.

In contrast to Kerner’s empirical analysis of the same freeway stretch, our study is based on an analysis of the spatio-temporal traffic dynamics rather than the dynamics at single freeway sections or comparisons of the dynamics at subsequent sections. The growth of small perturbations in the absence of ramps demonstrated by us is not compatible with the Lighthill-Whitham-Richard model, while several second-order models can reproduce them. Growing perturbations may be also described by first-order models with an additional diffusion term  $D(\rho) \partial^2 \rho / \partial x^2$ , which becomes negative in an unstable range at medium densities:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial Q(\rho(x, t))}{\partial x} = D(\rho(x, t)) \frac{\partial^2 \rho(x, t)}{\partial x^2} + \text{higher order terms} + \nu_+(x, t) - \nu_-(x, t). \quad (22)$$

Such models have common features with second-order models and can be theoretically supported (Nelson, 2000; Helbing, 2001a), but the stability of their solutions as a function of higher-order terms requires further investigations. In any case, the congested traffic states identified by us are in good agreement with predictions of some second-order macroscopic traffic models and some microscopic car-following models. Moreover, the dependence of the resulting traffic patterns on the respective freeway and ramp flows was qualitatively correct despite the variation of the flow data.

The erratic scattering of flow-density data is partly an effect of aggregation by loop detector data (i.e. the measurement procedure, see Helbing *et al.*, 2003; Treiber and Helbing, 2003). However, the main reason seems to be the broad distribution of the individual netto time gaps (Banks, 1999; Neubert *et al.*, 1999; Treiber and Helbing, 1999; Tilch and Helbing, 2000; Helbing *et al.*, 2002, 2003; Nishinari *et al.*, 2003). The variation of the density  $\rho$  and the average netto time gap  $T$  together can explain changes in the flow with a correlation of more the 90% (Nishinari *et al.*, 2003): According to Eq. (2), a variation of  $T$  causes a variation in the slope of the jam line  $J(\rho)$ , while variations of  $\rho$  cause variations along the respective jam line. Altogether this can explain variations perpendicular and parallel to the average jam line. A simulation of a mixture of cars and trucks had already pointed in this direction (Treiber and Helbing, 1999). In summary, an erratic and wide scattering of flow-density data can be reproduced, taking into account the heterogeneity in the driver-vehicle behavior due to the wide distribution of time gaps. Therefore, it does not contradict models with a fundamental diagram.

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